

PiXL Independence:

Mathematics - Student Booklet

KS4 HIGHER

Topic 1 – Decimals, Best Buys, Standard Form and Bounds.

Contents:

- I. Basic Skills Check
- II. Short Exam Questions
- III. Further Practice
- IV. Investigations
- V. Academic Stretch



I. Basic Skills check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator where possible.

Skills Check 1

1. A phone is reduced by 30% to a price of £120. Calculate the original cost of the system.



2. What is $\frac{2}{3}$ of £240?

3. Write 48 as a product of its prime factors in index form.

4. Factorise $x^2 - 2x - 35$.

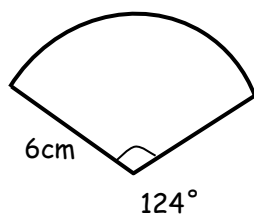
5. Write 210,000,000 in standard form.

6. Solve: $2x + 7 \leq -1$.

7. Use prime factors to find the lowest common multiple of 270 and 84.

8. List the first 5 terms of the sequence $8n + 2$.

9. Find the **total** perimeter of the sector shown, correct to one decimal place.

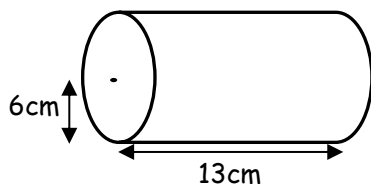


10. Calculate $(4.85 \times 10^{25}) \div (1.7 \times 10^{19})$, giving your answer in standard form correct to **two** significant figures.



Skills Check 2

1. A year ago Peter was 162 cm. He is now 3% taller. How tall is Peter now?
2. Calculate $(2.1 \times 10^{12}) \times (6.4 \times 10^{13})$, giving your answer in standard form correct to two significant figures.
3. Write 40 as a product of prime factors. What is the LCM of 40 and 54?
4. Factorise $x^2 - 20x + 75$.
5. Write 0.000000652 in standard form.
6. Solve: $5 \leq 6 - 3x$.
7. Calculate the total surface area of a cylinder with radius 6cm and length 13cm. Give your **final** answer to one decimal place.



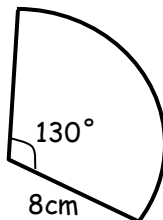
8. Find the n th term of the sequence: 15, 13, 11, 9,.....
9. Find the reciprocal of 3.5, giving your answer as a fraction.
10. A class of 14 boys and 18 girls take a test. The mean mark for the boys is 68%, and the mean mark for the girls is 84%. Calculate the mean mark for the whole class.






Skills Check 3

1. A shopkeeper buys small scooters at £32. She sells the bikes making 15% profit. Calculate the selling price of the scooters.
2. George uses $\frac{3}{4}$ of a tin of dog food each day. How many tins does he use in 5 days?
3. Find the HCF of 21 and 56.
4. Factorise $x^2 + x - 12$.
5. Write 0.00000004025 in standard form.
6. Solve: $3x + 7 \geq 5x - 2$.
7. Find the **total** perimeter of the sector shown, correct to one decimal place.



8. Find the n th term of the sequence: 2, 8, 18, 32,....
9. Use prime factors to find the highest common factor of 2520 and 924. 
10. A class of 12 boys and 18 girls take a test. The mean mark for the class is 70%. If the mean mark for the boys is 61%, find the mean mark for the girls.



II. Short Exam Questions

Section 1 Best Value & Exchange Rates

For each question you should attempt to carry out an estimation first.

1. Apples cost £1 for a 1.4 kg bag at Tesco. The same type of apples cost 87p for a 750g bag at ASDA. Where are the apples better value for money? You must show your working.
2. A pack of 9 kitchen rolls costs £4.19. A pack of 4 kitchen rolls costs £1.76. Which pack gives the better value for money? You must show all your working.
3. The exchange rate in London is £1 = €1.17. The exchange rate in Paris is €1 = £0.76. Janet wants to change some pounds into euros. In which of these cities would Janet get the most euros? You must show all your working.
4. Ann went to France. She changed £400 into Euros (€). The exchange rate was £1 = €1.34.
 - (a) How many euros did she get?
 - (b) Ann went shopping in France. She bought: 2 scarves for €3.40 each, 1 necklace for €16.40, 1 bag for €10.50. The exchange rate was £1 = €1.34.
Work out her total bill in pounds (£).

5. Sara is going to print 120 photos.
Here is some information about the cost of printing in two shops.

PHOTOBOX 15 prints £2.99.

DIGITAL WORLD 10 prints £3.49, buy one pack get one free.

Sara wants to buy the photos as cheaply as possible.

Which shop should Sara buy the 120 photos from?

You must show how you get your answer.



Section 2 - BIDMAS

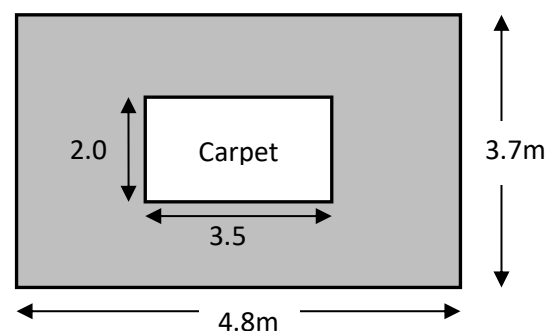
- Use 'BIDMAS' to find the value of these:
 - $3 + 4 \times 6$
 - $11 - 3 \times 2 + 5$
 - $(2^4 - 7) + (17 - 3^2)$
 - $\frac{7 + 11}{9}$
 - $\frac{27 - 11}{9 - 7}$
- Explain fully how you would use a mental method to find:
 - 7×0.2
 - $6 \div 0.2$
 - 40×0.025
 - $3.6 \div 0.06$
- Show all your working when solving these.
DO NOT USE A CALCULATOR.
 - $23.4 + 12.12$
 - $103.3 - 56.1$
 - $34 - 19.8$
 - 12.5×7
 - 47.2×1.6
 - $42 \div 0.7$
- Use LONG DIVISION to work this out.
DO NOT USE A CALCULATOR.
 $59.13 \div 2.7$
- Put brackets into the following expressions so that the answer is the number written in brackets.
e.g. $5 + 6 - 2$ (9)
Answer: $5 + (6 - 2) = 9$
 - $2 + 3 \times 4 - 1$ (19)
 - $2 + 3^2 \times 4 + 3$ (65)
 - $40 \div 10 \div 2$ (8)
 - $15.7 + 1.3 \times 8.7 + 1.3$ (170)
- Calculate the cost of 22.45kg of Strontium2007 at a cost of £176.52 per kg.
Give your answer to a sensible degree of accuracy.
- A bird travels 85 m (to the nearest m) in 12 seconds (to the nearest second).
Calculate its MAXIMUM possible average speed.
- A tanker can hold up to 41000 litres of petrol, correct to 3sf. Steel barrels can hold up to 660 litres, correct to the nearest 10 litres. Mark begins emptying a full tanker into barrels.
What is the least number of barrels needed so that he can be certain that all the petrol in the tanker can be emptied into the barrels?



Section 3 - Standard Form

- Use prime factors to find the HCF and LCM of 504 and 216.
- Use your calculator to find each of these. Give answers in standard form to 3 significant figures:
 - $(3.85 \times 10^{13}) \times (1.63 \times 10^{11})$
 - $\sqrt{(6.53 \times 10^{-7})}$
- Without a calculator, and showing your working, find each of these in standard form:
 - $(9.2 \times 10^{20}) \times (2 \times 10^{30})$
 - $(3 \times 10^{13}) \div (5 \times 10^{-17})$
- Assuming that the Earth's orbit round the Sun is approximately a circle of radius 1.50×10^{11} metres, and that it orbits the Sun in exactly one year, find its speed in metres per second.
Give your answer in standard form to 3 significant figures.
- A pollen grain has a mass of 3.5×10^{-4} grams. Find the number of grains in 1kg of pollen. Write your answer in standard form to **two** significant figures.
- Use your calculator to work these out. Give your answers in standard form to three significant figures.
 - $(3.82 \times 10^{23}) \div (5.1 \times 10^{-12})$
 - $\sqrt{(1.8 \times 10^{37})}$
 - $(2.4 \times 10^{-5}) + (1.29 \times 10^{-6})$
 - The **reciprocal** of 6.5×10^{-5} .
- There are 14700 (to the nearest hundred) spectators at a football match.
 - What is the minimum possible number of spectators?
 - What is the maximum possible number of spectators?
- This diagram shows the plan of a room.
All the dimensions, including the carpet, are estimated to the nearest 0.1 metres.
Calculate the maximum possible shaded area.

7



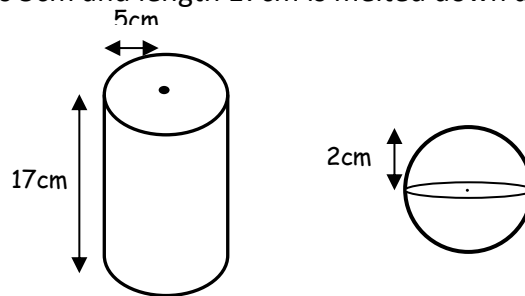
Section 4 - Mixed Questions

1. A one ounce measure of poppy seeds contains approximately 1.4×10^5 seeds.

Given that one ounce is equal to 28.3 g, work out how many poppy seeds would be in a 1 kg measure of seeds.

Give your answer in standard form correct to 2 significant figures.

2. A solid metal cylinder of radius 5cm and length 17cm is melted down and made into spheres of radius 2cm.



Assuming that none of the metal is lost in the process, work out how many whole spheres can be made.

3. There are 500 sheets, correct to the nearest 10, in a pile of paper. The height of the pile is 46mm, correct to the nearest mm. Work out the upper bound for the thickness of one sheet of paper.

Give your answer correct to 2 significant figures.



4. In this question, assume that the planets move in circular orbits in the same plane with the Sun (S) at the centre.

P is a point on the Earth's orbit.

The radius of the Earth's orbit is 9.30×10^7 miles.

The radius of the orbit of Mars is 1.42×10^8 miles.



- a) (i) Find the distance of the point on the orbit of Mars that is furthest from P. Give your answer in standard form.
- (ii) Find the distance of the point on the orbit of Mars that is closest to P. Give your answer in standard form.
- b) (i) Work out the circumference of the circular orbit of Earth. Give your answer as an ordinary number correct to 3 significant figures.
- (ii) The Earth orbits the Sun in one year. Work out the speed of the Earth, in miles per hour, as it orbits the Sun.
5. Showing all your working, estimate the value of 0.287×0.513 .
6. A rectangle has an area of 24.3cm^2 and a length of 6.2cm , both given correct to one decimal place. Find upper and lower bounds for its width.
7. A car travels 465 metres (to the nearest 5m) in a time of 17.0 seconds (to the nearest tenth of a second). Find correct to **two decimal places** the upper bound for its average speed over this time.
- [If time, convert this speed to miles per hour, given that 1 mile = 1609.34 metres]



III. Further Practice

1. Can you solve the murder?
<https://justmaths.co.uk/Worksheets/Number/AA%20NEW%20-%20Estimating%20to%201%20sigfig%20-%20Whodunnit.pdf>
<https://justmaths.co.uk/Worksheets/Number/Bidmas%20-%20Whodunnit.pdf>
2. Watch and make notes, answer any questions as you go.
<https://www.youtube.com/watch?v=0W0C3KQDyVg>
3. Read the information and try to answer the questions. Can you think how you might extend this investigation further?
<https://justmaths.co.uk/Worksheets/Number/Functional%20Skills%20-%20EXCHANGE%20RATES.pdf>
4. Follow the 'revise, activity, test'.
<http://www.bbc.co.uk/schools/gcsebitesize/maths/number/roundestimaterev1.shtml>
5. Answer the exam questions. Then mark your answers.
http://www.mathsgenie.co.uk/resources/82_standard-form.pdf
http://www.mathsgenie.co.uk/resources/82_standard-formans.pdf
6. Watch the revision video then answer the questions and mark your answers.
<http://www.mathsgenie.co.uk/bounds.html>
http://www.mathsgenie.co.uk/resources/97_upper-and-lower-bounds.pdf
http://www.mathsgenie.co.uk/resources/97_upper-and-lower-boundsans.pdf
7. Watch the video, then answer all the exam questions and mark your answers.
https://www.youtube.com/watch?v=75KeV8_vlFs



IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

1. Pick one of the investigations in the booklet and answer the questions. In order to gain full credit you need to extend your investigation further. Explain at each stage the decisions you have made and any changes in your approach. Your investigation will need a detailed summary.

<http://social.ocr.org.uk/files/ocr/Maths%20investigations.pdf>

2. NRICH activities. Follow the instructions, can you extend your thinking further? Follow some of the links? Keep notes of what you are doing at each stage.

<https://nrich.maths.org/13325>

<https://nrich.maths.org/7024>

<https://nrich.maths.org/746>

3. Exploring maths.

<https://nrich.maths.org/7016>

<https://wild.maths.org/spirals-spirals>

<https://wild.maths.org/quarters-quarters>

4. **Can you find a sequence of consecutive integers that add up to 1000?** Try this investigation first, then read the following solution and make notes on the methods. What would you do differently? What did you change each time?

<https://ibmathsresources.com/2015/04/10/arithmetic-sequences-puzzle/>

5. **Investigate what makes numbers happy, or sad.**

<https://ibmathsresources.com/2014/06/19/friendly-numbers-solitary-numbers-perfect-numbers/>

<https://nrich.maths.org/1314/index>



V. Academic Reading

Maths in the real world.

For each video or article you should make notes and questions you would like answering to extend your understanding and knowledge of maths in the real world.

1. Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'

Where do maths symbols come from?

<https://ed.ted.com/lessons/where-do-math-symbols-come-from-john-david-walters>

2. Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'

Banned numbers.

<https://ed.ted.com/lessons/a-brief-history-of-banned-numbers-alessandra-king>

3. Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'

Maths anxiety.

<https://ed.ted.com/lessons/why-do-people-get-so-anxious-about-math-orly-rubinsten>

**4. Read the following article, can you produce a poster, or presentation using this article?
Can you research any of these elements further?**

<https://www.theguardian.com/science/2012/jan/23/how-learn-love-maths>

5. Japanese Multiplication.

Watch the video then design instructions to explain how this works.

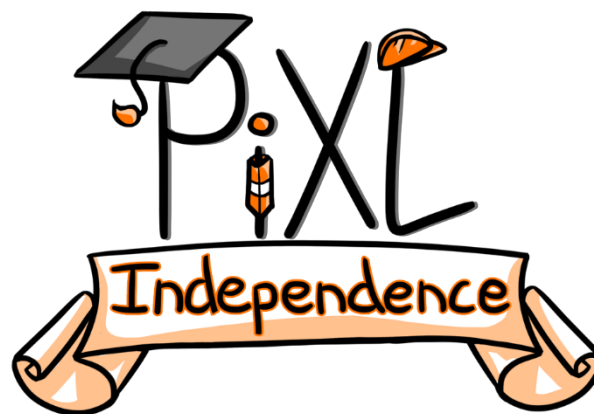
https://www.youtube.com/watch?time_continue=94&v=85Vd0NpL32k

6. Pick one subject from the list and follow the materials and questions to extend your understanding about mathematics in other subjects.

<https://integralmaths.org/course/view.php?id=166>

7. Read one of the articles from the link and make detailed notes about how standard form is used.

<http://www.jodrellbank.manchester.ac.uk/>



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