GCSE Mathematics Foundation Tier WJEC

Number · Algebra · Geometry & measures · Statistics

Here is pretty much all the Foundation Tier content we could fit onto an A3 sheet of paper, including all the formulae you are required to know for GCSE. An ➔ points to an illustrative example. NM = Numeracy and GCSE Mathematics; M = GCSE Mathematics only. Pin this to a wall, keep it on your desk, carry it in your bag, make notes on it (sorry, don't take it into the examination)…

BIDMAS (NM)

…or BODMAS. Use the correct order of operations; take care when using a calculator.

- **B**rackets
- **I**ndices (or p**O**wers)
- **D**ivision and **M**ultiplication
- **A**ddition and **S**ubtraction

Directed numbers (NM)

Use the rules "two negatives make a positive", "one of each make a negative"…

Addition and subtraction

➔ $-5 + -7$
$-5 - 7 = -12$

Multiplication or division

➔ -5×-7
$-5 \times -7 = +35$

Types of number (NM)

Integer: a "whole" number
Factors: the divisors of an integer
➔ Factors of 12 are 1, 2, 3, 4, 6, 12
Multiples: a "times table" for an integer (will continue indefinitely)
➔ Multiples of 12 are 12, 24, 36 …
Prime number: an integer which has exactly two factors (1 and the number itself). Note: 1 is not a prime number.

Prime factors (NM)

Write a number as a product of its prime factors; use indices for repeated factors:
➔ $720 = 5 \times 3^2 \times 2^4$

Standard form (NM)

Standard form numbers are of the form $a \times 10^n$ where $1 \le a < 10$ and n is an integer.

Decimal places (NM)

Truncate the number, then use a "decider digit" to round up or down. Count digits from the decimal point
➔ 162.3681 to 2dp;
162.36 | 81 = 162.37 to 2dp

Estimation (NM)

Round each number in a calculation so that it is easier to work out (even though the answer will not be exact)
➔ Estimate the value of $\frac{38 \times 217}{52}$
Rounding off gives
$\frac{40 \times 200}{50} = 8000 \div 50 = 800 \div 5$
Estimated value is 160

Division using ratio (NM)

Use a ratio for unequal sharing
➔ Divide £480 in the ratio 7 : 5
7 + 5 = 12, then £480 ÷ 12 = £40
7 × £40 = £280, 5 × £40 = £200
(check: £280 + £200 = £480 ✓)

Ratio and fractions (NM)

Link between ratios and fractions
➔ Boys to girls in ratio 2 : 3
$\frac{2}{5}$ are boys, $\frac{3}{5}$ are girls.

Calculating with fractions (NM)

Adding or subtracting fractions; use a common denominator…
➔ $\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$
Multiplying fractions; multiply numerators and denominators…
➔ $\frac{4}{7} \times \frac{2}{3} = \frac{8}{21}$
Dividing fractions; "flip" the second fraction, then multiply…
➔ $\frac{2}{7} \div \frac{5}{6} = \frac{2}{7} \times \frac{6}{5} = \frac{12}{35}$
Fraction of an amount; divide and multiply…
➔ $\frac{3}{5}$ of $70 = 70 \div 5 \times 3 = 42$

Improper fractions (NM)

[note: an improper fraction is often called a "top heavy" fraction]
➔ Change $\frac{25}{7}$ to a mixed number
$25 \div 7 = 3$ with remainder 4
so $\frac{25}{7} = 3\frac{4}{7}$
➔ Change $5\frac{2}{9}$ to an improper fraction
$5 \times 9 + 2 = 47$
so $5\frac{2}{9} = \frac{47}{9}$

Fractions, decimals (NM)

Fraction is numerator ÷ denominator
➔ $\frac{5}{8} = 5 \div 8 = 0.625$
Use place values to change decimals to fractions. Simplify where possible.
➔ $0.45 = \frac{45}{100} = \frac{9}{20}$
Learn the most frequently used ones:

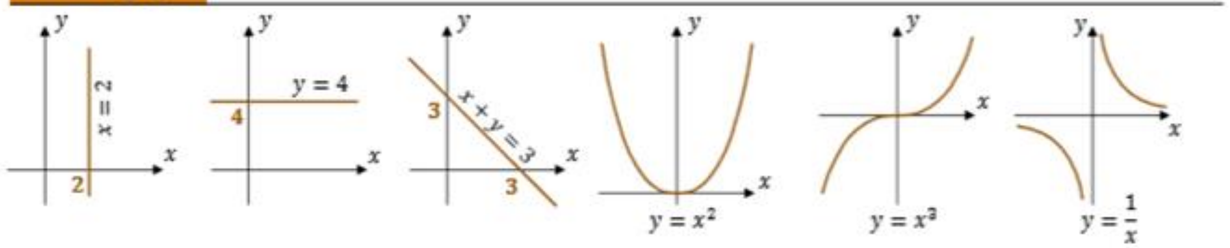
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{4}$
0.5	0.25	0.1	0.2	0.75

Percentages (NM)

y percent of $x = \frac{y}{100} \times x$
➔ Increase £58 by 26%.
$\frac{26}{100} \times £58 = £15.08$
£58 + £15.08 = £73.08
y as a percentage of $x = \frac{y}{x} \times 100\%$
➔ The population of a town increases from 3500 to 4620
Find the percentage increase.
$\frac{1120}{3500} \times 100\% = 32\%$
Note: fraction $= \frac{\text{increase}}{\text{original}}$
Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{100}$
50%	25%	10%	20%	1%

Standard graphs (M)



Algebraic notation (NM)

$ab = a \times b$
$3y = y + y + y$
$a^2 = a \times a$
$a^3 = a \times a \times a$
$a^2b = a \times a \times b$
$\frac{a}{b} = a \div b$

Expanding brackets (M)

$p(q + r) = pq + pr$
➔ $5(x - 2y) = 5x - 10y$

Sequences (NM)

Triangular numbers:

1st	2nd	3rd	4th	5th
1	3	6	10	15

Square numbers ($n^2 = n \times n$):

1^2	2^2	3^2	4^2	5^2
1	4	9	16	25

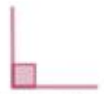
Cube numbers ($n^3 = n \times n \times n$):

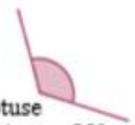
1^3	2^3	3^3	4^3	5^3
1	8	27	64	125

nth term of an arithmetic (linear) sequence is $an + d$
➔ nth term of 5, 8, 11, 14, … is $3n+2$ (always increases by 3 first term is $3 \times 1 + 2 = 5$)

Types of angle (NM)


Acute [less than 90°]


Right angle [exactly 90°]

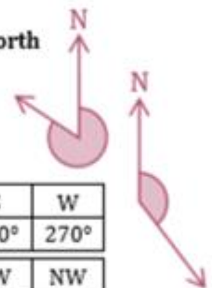

Obtuse [between 90° and 180°]


Reflex [between 180° and 360°]

Bearings (NM)

Always measured **clockwise from North**

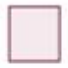
Always use three digits [for example 38° is written 038°]



N	E	S	W
000°	090°	180°	270°

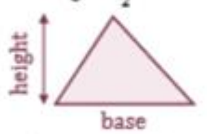
NE	SE	SW	NW
045°	135°	225°	315°

Quadrilaterals (G20, G22)

		side lengths	angles	symmetry	parallel sides
	kite	2 equal pairs	1 equal pair	1 line	none
	parallelogram	2 equal pairs	2 equal pairs	no lines; rotational order 2	2 pairs
	rectangle	2 equal pairs	all equal 90°	2 lines; rotational order 2	2 pairs
	rhombus	all equal	2 equal pairs	2 lines; rotational order 2	2 pairs
	square	all equal	all equal 90°	4 lines; rotational order 4	2 pairs
	trapezium	-	2 pairs that total 180°	-	1 pair

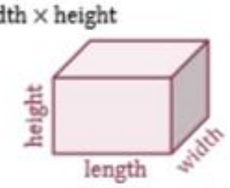
Areas and volumes (NM)

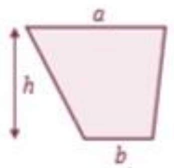
Area of triangle $= \frac{1}{2} \times$ base $\times$ height



Circumference of circle $= \pi \times D$
Area of circle $= \pi \times r^2$

Volume of cuboid = length × width × height





Area of trapezium $= \frac{1}{2}(a + b) \times h$

Transformations (M)

Rotation
- Centre of rotation
- Angle of rotation
- Clockwise or anticlockwise

Reflection
- Line of reflection

Translation
- Horizontal
- Vertical

Enlargement
- Centre of enlargement
- Scale factor

Angle facts (NM)

Angles in parallel lines: always use correct terminology

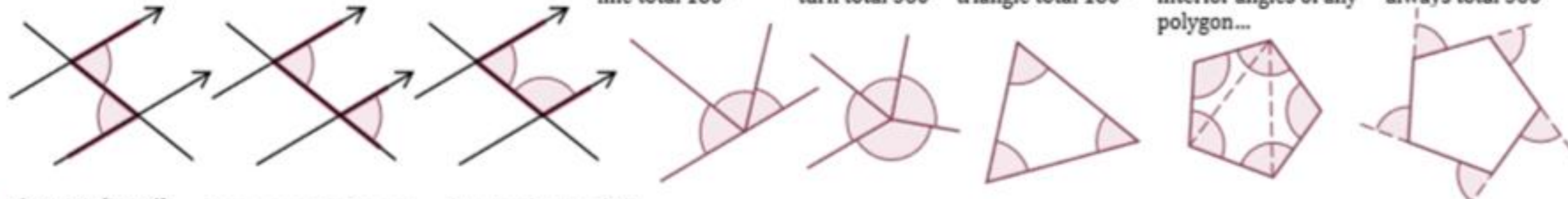
Angles on a straight line total 180°

Angles in a full turn total 360°

Interior angles in a triangle total 180°

Use triangles for the interior angles of any polygon…

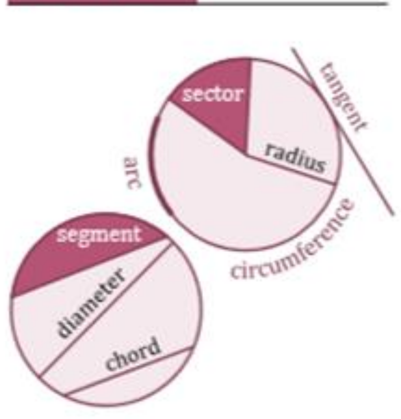
Exterior angles always total 360°



Alternate [equal] · Corresponding [equal] · Interior [total 180°]

…or $180° \times (n - 2)$

Parts of a circle (NM)



Standard units (NM)

1 tonne	= 1000 kilograms
1 kilogram	= 1000 grams
1 kilometre	= 1000 metres
1 metre	= 100 centimetres
	= 1000 millimetres
1 centimetre	= 10 millimetres
1 day	= 24 hours
1 hour	= 60 minutes
	= 3600 seconds
1 minute	= 60 seconds

Metric - imperial conversions (NM)

8 kilometres	≈ 5 miles
1 kilogram	≈ 2.2 pounds
1 litre	≈ 1.75 pints

LEARN THESE!

➔ I am driving at 35mph. The speed limit is 50kph. Am I breaking the speed limit?
35 ÷ 5 = 7
7 × 8 = 56 kilometres
Yes I am breaking the speed limit

Speed, distance, time (NM)

Speed $= \frac{\text{distance}}{\text{time}}$
➔ A car travels 90 miles in 1 hour, 30 minutes. Find its average speed.
90 miles ÷ 1.5 hours = 60 mph

Currency conversion (NM)

➔ A camera costs £450 in London
The camera costs $560 in New York
The exchange rate is £1 = $1.30
Where is the camera cheaper?
450 × 1.30 = $585
Camera is cheaper in New York

Averages (NM)

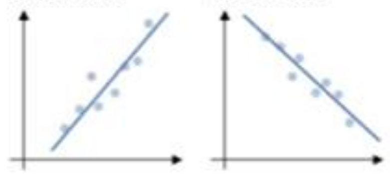
Mode: most frequently occurring
Median: put the data in numerical order, then choose the middle one
Mean $= \frac{\text{total of items of data}}{\text{number of items of data}}$

Measure of spread (NM)

Range = maximum − minimum

Correlation (NM)

Positive correlation · Negative correlation



Probability (M)

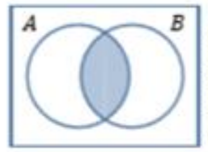
$p = \frac{n(\text{equally likely favourable outcomes})}{n(\text{equally likely possible outcomes})}$

$p = 0$	impossible
$0 < p < 0.5$	unlikely
$p = 0.5$	evens
$0.5 < p < 1$	likely
$p = 1$	certain

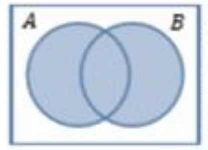
P(event does not occur) = 1 − P(event does occur)

Total probability of all the possible outcomes of an experiment is 1

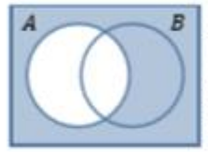
Venn diagrams (NM)



$A \cap B$
Intersection of A and B. Elements in A **and** in B



$A \cup B$
Union of A and B
Elements in A **or** B **or** both



A'
Complement of A
Elements which are **not** in A

GCSE Mathematics Intermediate Tier WJEC

● Number ● Algebra ● Geometry & measures ● Statistics

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Percentages — NM

Increase or decrease by a percentage.
➔ Increase £45 by 8%.

$\frac{8}{100} \times 45 = 3.6$

$45 + 3.6 = £48.60$

Express as a percentage
➔ What is a mark of 36 out of 78 as a percentage?

$\frac{36}{78} \times 100 = 46.2\%$ (1dp)

Percentage profit and loss — NM

Express the profit or loss as a percentage of the **original** amount.
➔ I buy an antique for £250 and sell it for £420. What is my percentage profit?

$\frac{420-250}{250} \times 100 = 68\%$

Reverse percentage — NM

Don't find percentage of new amount
➔ The price of a jacket is reduced by 35% to £156. What was the original price of the jacket?
[note that £156 is 65% of the original]

65% = £156

1% = £156 ÷ 65 = £2.40

Original price is 100 × £2.40 = £240

Compound interest — NM

Total accrued $= P\left(1 + \frac{r}{100}\right)^n$

➔ I invest £600 at 3% compound interest. What is my account worth after 5 years?

$£600 \times \left(1 + \frac{3}{100}\right)^5 = £695.56$

Error intervals — NM

Find the range of numbers that will round to a given value:
➔ $x = 5.83$ (2 decimal places)
$5.825 \le x < 5.835$
➔ $y = 46$ (2 significant figures)
$45.5 \le y < 46.5$
Note use of ≤ and <, and that the last significant figure of each is 5

If numbers are added or subtracted, **add** the error intervals.
➔ $a = 6.3$ and $b = 2.5$ (1dp)
$a - b = 6.3 - 2.5 \pm (0.05 + 0.05)$
$a - b = 3.8 \pm 0.1$

Powers and roots — NM

Special indices: for any value a:

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$a^{\left(\frac{1}{n}\right)} = \sqrt[n]{a}$

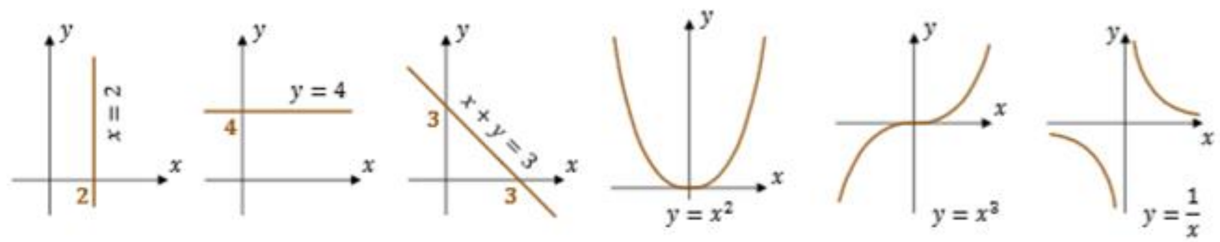
➔ $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

➔ $8^{\left(\frac{1}{3}\right)} = \sqrt[3]{8} = 2$

Standard form — NM

Numbers of the form $a \times 10^n$, where where $1 \le a < 10$ and n is an integer.

Standard graphs — M



Laws of indices — NM

For any value a:

$a^x \times a^y = a^{x+y}$

$\frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{xy}$

➔ $\frac{x^8y^3}{x^2y} = x^6y^2$

y = mx + c — M

Equation of straight line $y = mx + c$
m is the gradient; c is the y intercept:
➔ Find the equation of the line that joins $(0,3)$ to $(2,11)$
Find its gradient...

$\frac{11-3}{2-0} = \frac{8}{2} = 4$

...and its y intercept...
Passes through $(0,3)$, so c = 3
Equation is $y = 4x + 3$

Parallel, perpendicular lines — M

Parallel lines: gradients are equal; perpendicular lines: gradients are "negative reciprocals".
➔ $y = 2x + 3$ and $y = 2x - 5$ are parallel to each other; $y = 2x + 3$ and $y = -\frac{1}{2}x + 3$ are perpendicular

Sequences — NM

Triangular numbers:

1st	2nd	3rd	4th	5th
1	3	6	10	15

Square numbers ($n^2 = n \times n$):

1^2	2^2	3^2	4^2	5^2
1	4	9	16	25

Cube numbers ($n^3 = n \times n \times n$):

1^3	2^3	3^3	4^3	5^3
1	8	27	64	125

nth term of an arithmetic (linear) sequence is $an + d$
➔ nth term of 5, 8, 11, 14, ... is $3n+2$ (always increases by 3 first term is $3 \times 1 + 2 = 5$)

Brackets — M

Expand: multiply out brackets...
$(x + a)(x + b) = x^2 + ax + bx + ab$
➔ $(2x - 3)(x + 5)$
$= 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$

Factorise: put into brackets...
➔ Factorise fully $10x^3 + 8xy^2$
$2x(5x^2 + 4y^2)$
➔ Factorise $x^2 - 3x - 28$
$(x - 7)(x + 4)$

Simultaneous equations — M

➔ Solve $\begin{cases} 2x + 3y = 11 \\ 3x - 5y = 7 \end{cases}$

Multiply to match a term in x or y

$\begin{cases} 10x + 15y = 55 \\ 9x - 15y = 21 \end{cases}$

Add or subtract to cancel...
$19x = 76$, so $x = 4$
Finally, substitute and solve...
$2 \times 4 + 3y = 11$, so $y = 1$

Trial and improvement — M

➔ Solve $x^3 + 2x = 250$ to 1dp, given that $6 < x < 7$
Trial a value (say 6.5) with $6 < x < 7$
$6.5^3 + 2 \times 6.5 = 287.6$ ... (too low)
Find two consecutive values...
$6.1^3 + 2 \times 6.1 = 239.2$ ... (too low)
$6.2^3 + 2 \times 6.2 = 250.7$ ... (too high)
Test intermediate value...
$6.15^3 + 2 \times 6.15 = 244.9$ ... (too low)
Hence $6.15 < x < 6.2$
...so solution is closer to 6.2 than 6.1
$x = 6.2$ (to 1dp)

Quadratics — M

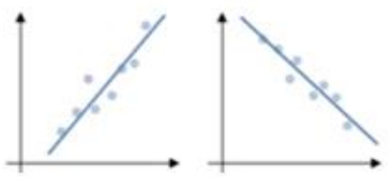
Solve a quadratic by factorising.
➔ Solve $x^2 + 8x + 15 = 0$
Put into brackets (taking care with negative numbers)...
$(x + 5)(x + 3) = 0$
...then either $x + 5 = 0$ or $x + 3 = 0$ so that $x = -5$ or $x = -3$

Rearrange a formula — M

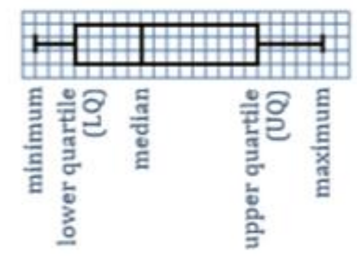
The subject of a formula is the term on its own. Use rules that "balance" the formula to change its subject
➔ Make x the subject of
$2x + 3y = z$
Here, subtract 3y from both sides...
$2x = z - 3y$
...then divide both sides by 2
$x = \frac{z - 3y}{2}$

Correlation — NM

Positive correlation / Negative correlation

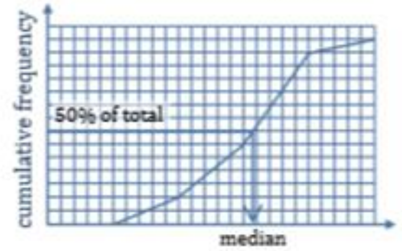


Box plots — NM



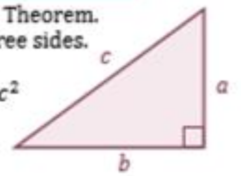
Cumulative frequency — NM

Plot points at the **upper end** of each interval. May not start at $(0, 0)$



Right angled triangles — NM

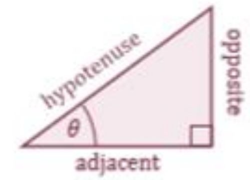
Pythagoras Theorem.
Links all three sides.
No angles.
$a^2 + b^2 = c^2$



Trigonometry.
Links two sides and one angle.
SOH | CAH | TOA

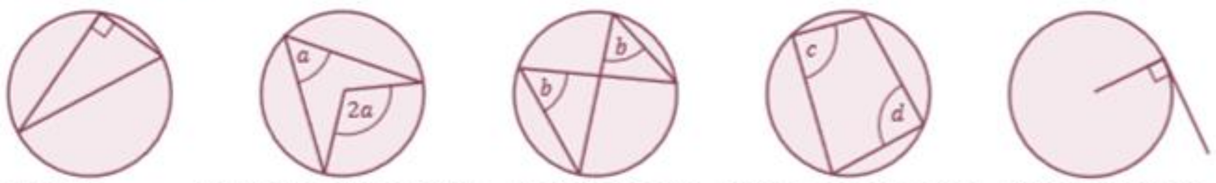
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Use "2ndF" or "SHIFT" key to find a missing angle



The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Circle theorems — M



Angle in a semicircle is 90° | Angle at the centre is double the angle at the circumference | Angles in the same segment are equal | Opposite angles in a cyclic quadrilateral total 180° | Tangent and radius are perpendicular

Areas and volumes — NM

Area of triangle $= \frac{1}{2} \times$ base $\times$ height

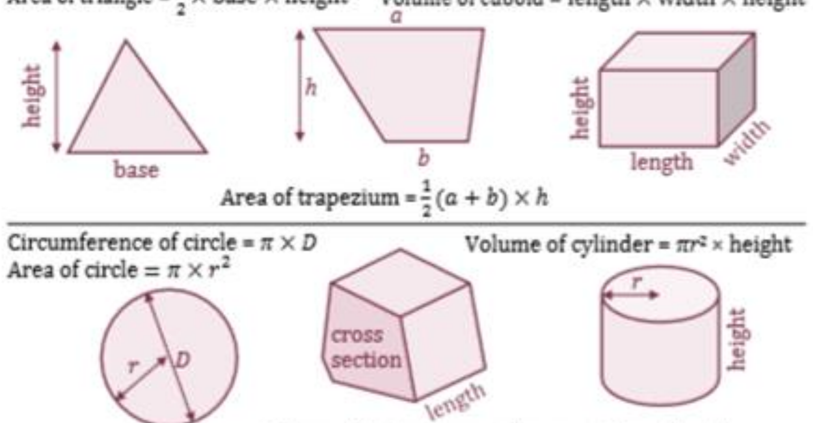
Volume of cuboid = length × width × height

Area of trapezium $= \frac{1}{2}(a + b) \times h$

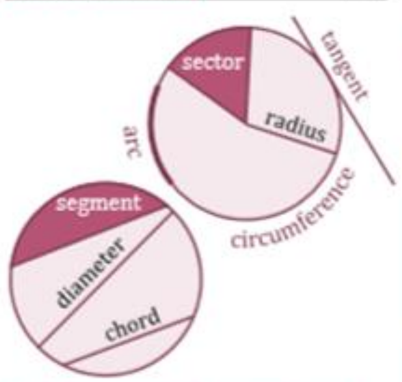
Circumference of circle $= \pi \times D$
Area of circle $= \pi \times r^2$

Volume of cylinder $= \pi r^2 \times$ height

Volume of prism = area of cross section × length



Parts of a circle — NM



Metric - imperial conversions — NM

8 kilometres ≈ 5 miles
1 kilogram ≈ 2.2 pounds
1 litre ≈ 1.75 pints

LEARN THESE!

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Transformations — M

Reflection
• Line of reflection
Translation
• Vector

Rotation
• Centre of rotation
• Angle of rotation
• Clockwise or anticlockwise

Enlargement
• Centre of enlargement
• Scale factor (if SF < 1 the shape will get smaller).

Angle facts — NM

Angles in parallel lines: always use correct terminology

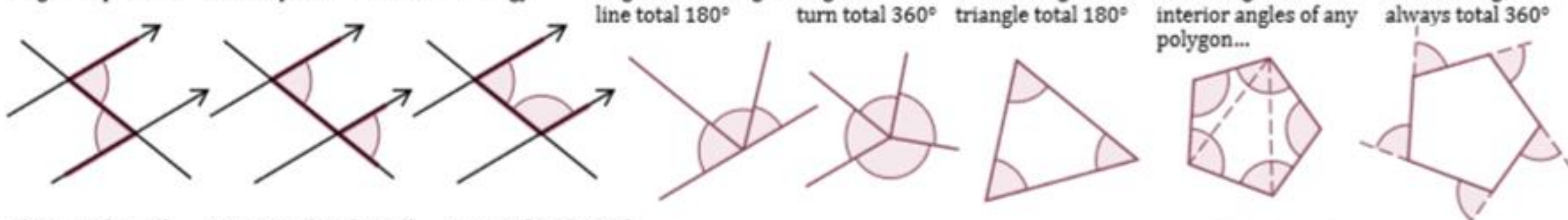
Angles on a straight line total 180°

Angles in a full turn total 360°

Interior angles in a triangle total 180°

Use triangles for the interior angles of any polygon...

Exterior angles always total 360°



Alternate [equal] Corresponding [equal] Interior [total 180°]

...or 180° × (n − 2)

Measures of spread — NM

Range = maximum − minimum
Interquartile range (IQR) = UQ − LQ

Averages — NM

Mode: most frequently occurring
Median: put the data in numerical order, then choose the middle one

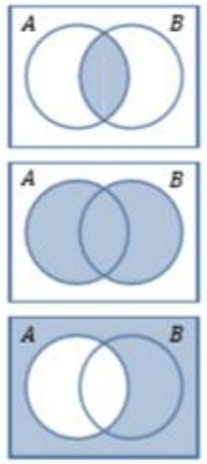
Mean $= \frac{\text{total of items of data}}{\text{number of items of data}}$

Tabulated data

x	f	$f \times x$
7	17	$7 \times 17 = 119$
8	9	$8 \times 9 = 72$
9	4	$9 \times 4 = 36$
Total	30	227

Mean = 227 ÷ 30 = 7.57 (2dp)
Mode = 7 [has the highest frequency]
[Note: if data is grouped, eg $5 < x \le 10$, etc, use the mid interval values]

Venn diagrams — NM



$A \cap B$
Intersection of A and B.
Elements in A **and** in B

$A \cup B$
Union of A and B
Elements in A **or** B **or** both

A'
Complement of A
Elements which are **not** in A

Probability rules — M

Multiply for independent events
P(A and B) = P(A) × P(B) [AND rule]
➔ P(6 on dice and H on coin)

$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Add for mutually exclusive events
P(A or B) = P(A) + P(B) [OR rule]
➔ P(5 or 6 on dice)

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Apply these rules to tree diagrams

GCSE Mathematics Higher Tier WJEC

● Number ● Algebra ● Geometry & measures ● Statistics

Here is pretty much all the Higher Tier content we could fit onto an A3 sheet of paper, including all the formulae you are required to know for GCSE. An ➔ points to an illustrative example. NM = Numeracy and GCSE Mathematics; M = GCSE Mathematics only. Pin this to a wall, keep it on your desk, carry it in your bag, make notes on it (sorry, don't take it into the examination)...

Recurring decimals NM

Make a recurring decimal a fraction:

➔ $n = 0.2\dot{3}\dot{6}$

(two digits are in the recurring pattern, so multiply by 100)

$100n = 23.6\dot{3}\dot{6}$

(this is the same as $23.63\dot{6}$)

$99n = 23.636 - 0.236 = 23.4$

$n = \frac{23.4}{99} = \frac{234}{990} = \frac{13}{55}$

Percentages: multipliers NM

Percentage increase or decrease; use a multiplier (powers for repetition)

➔ **Initially there were 20 000 fish in a lake. The number decreases by 15% each year. Estimate the number of fish after 6 years.**

$20\,000 \times 0.85^6 = 7500$ (2sf)

➔ **The price of a jacket is reduced by 35% to £156. What was the original price of the jacket?**

[note that £156 is 65% of the original]

£156 ÷ 0.65 = £240

Compound interest NM

Total accrued $= P\left(1 + \frac{r}{100}\right)^n$

➔ **I invest £600 at 3% compound interest. What is my account worth after 5 years?**

$£600 \times \left(1 + \frac{3}{100}\right)^5 = £695.56$

Annual Equivalent Rate NM

Annual Equivalent Rate, as a decimal,

$\text{AER} = \left(1 + \frac{i}{n}\right)^n - 1$

➔ **The nominal interest rate *per annum* is 5%. It is paid each month. Find the AER as a percentage.**

$\left(1 + \frac{0.05}{12}\right)^{12} - 1 = 0.051\,161\ldots$

AER is 5.12% (3sf)

Error intervals NM

Find the range of numbers that will round to a given value:

➔ $x = 5.83$ (2 decimal places)

$5.825 \le x < 5.835$

➔ $y = 46$ (2 significant figures)

$45.5 \le y < 46.5$

Note use of ≤ and <, and that the last significant figure of each is 5

Direct & inverse proportion NM

y is directly proportional to x

$y = kx$ for a constant k

➔ **b is directly proportional to a^2**

$a = 6$ when $b = 90$ Find b if $a = 8$

$b = ka^2$ $a = 6$ and $b = 90$ for k

$90 = k \times 6^2$ so $k = 2.5, b = 2.5a^2$

$b = 2.5 \times 8^2 = 160$

y is inversely proportional to x

$yx = k$ or $y = \frac{k}{x}$ for a constant k

Surds NM

Look for the biggest square number factor of the number:

➔ $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

Powers and roots NM

Special indices: for any value a:

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$a^{\left(\frac{p}{q}\right)} = \sqrt[q]{a^p}$

➔ $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

➔ $8^{\left(\frac{2}{3}\right)} = \sqrt[3]{8^2} = 4$

Standard form NM

Numbers of the form $a \times 10^n$, where where $1 \le a < 10$ and n is an integer.

Laws of indices NM

For any value a:

$a^x \times a^y = a^{x+y}$

$\frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{xy}$

➔ $\left(\frac{2pq^4}{p^3q}\right)^3 = \frac{8p^3q^{12}}{p^9q^3} = \frac{8q^9}{p^6}$ or $8q^9p^{-6}$

Rearrange a formula M

The subject of a formula is the term on its own. Rearrange to...

➔ **Make x the subject of**

$2x + ay = y - bx$

$2x + bx = y - ay$

$x(2 + b) = y - ay$

$x = \frac{y - ay}{2 + b}$

y = mx + c M

Equation of straight line $y = mx + c$

m is the gradient; c is the y intercept:

➔ **Find the equation of the line that joins $(0, 3)$ to $(2, 11)$**

Find its gradient...

$\frac{11 - 3}{2 - 0} = \frac{8}{2} = 4$

...and its y intercept...

Passes through $(0, 3)$, so c = 3

Equation is $y = 4x + 3$

Parallel, perpendicular lines M

Parallel lines: gradients are equal; perpendicular lines: gradients are "negative reciprocals".

➔ **$y = 2x + 3$ and $y = 2x - 5$ are parallel to each other; $y = 2x + 3$ and $y = -\frac{1}{2}x + 3$ are perpendicular**

Velocity - time graph NM

Gradient = acceleration (you may need to draw a tangent to the curve at a point to find the gradient);

Area under curve = distance travelled

Equations and identities M

An equation is true for some particular value of x

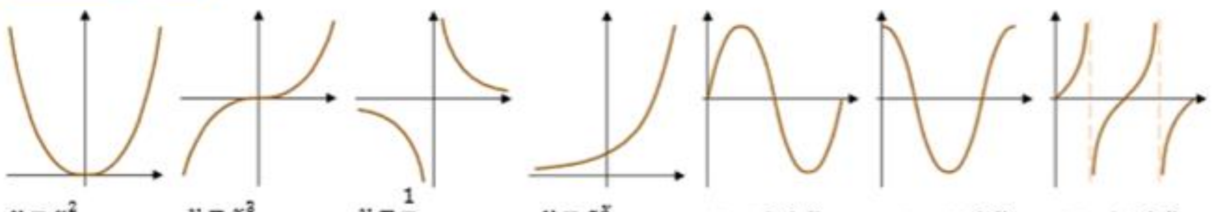
➔ $2x + 1 = 7$ **is true if** $x = 3$

...but an identity is true for every value of x

➔ $(x + a)^2 \equiv x^2 + 2ax + a^2$

(note the use of the symbol ≡)

Standard graphs M



$y = x^2$ $y = x^3$ $y = \frac{1}{x}$ $y = a^x$ $y = \sin(x°)$ $y = \cos(x°)$ $y = \tan(x°)$

Quadratics M

Solve a quadratic by factorising.

➔ **Solve $2x^2 - x - 10 = 0$**

Put into brackets (taking care with negative numbers or fractions)...

$(2x - 5)(x + 2) = 0$

...then either $2x - 5 = 0$ or $x + 2 = 0$

so that $x = 2\frac{1}{2}$ or $x = -2$

If a quadratic equation cannot be factorised (look for "round to 3sf", etc, in the question), use the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

➔ **Solve $2x^2 + 3x - 7 = 0$**

$x = \frac{-3 - \sqrt{9 - (-56)}}{2 \times 2} = -2.73$

or $x = \frac{-3 + \sqrt{9 - (-56)}}{2 \times 2} = 1.23$

Difference of two squares M

$a^2 - b^2 = (a + b)(a - b)$

➔ $x^2 - 25 = (x + 5)(x - 5)$

Sequences NM

nth term of an arithmetic (linear) sequence is $bn + c$

➔ ***n*th term of 5, 8, 11, 14, ... is $3n+2$ (always increases by 3 first term is $3 \times 1 + 2 = 5$)**

nth term of a quadratic sequence is $an^2 + bn + c$

➔ **First three terms of $n^2 + 3n - 1$ are 3, 9, 17, ...**

Simultaneous equations M

➔ Solve $\begin{cases} 2x + 3y = 11 \\ 3x - 5y = 7 \end{cases}$

Multiply to match a term in x or y

$\begin{cases} 10x + 15y = 55 \\ 9x - 15y = 21 \end{cases}$

Add or subtract to cancel...

$19x = 76$, so $x = 4$

Finally, substitute and solve...

$2 \times 4 + 3y = 11$, so $y = 1$

Trial and improvement M

➔ **Solve $x^3 + 2x = 250$ to 1dp, given that $6 < x < 7$**

Trial a value (say 6.5) with $6 < x < 7$

$6.5^3 + 2 \times 6.5 = 287.6$ ... (too low)

Find two consecutive values...

$6.1^3 + 2 \times 6.1 = 239.2$ ... (too low)

$6.2^3 + 2 \times 6.2 = 250.7$ ... (too high)

Test intermediate value

$6.15^3 + 2 \times 6.15 = 244.9$ ... (too low)

Hence $6.15 < x < 6.2$

$x = 6.2$ (to 1dp)

Transformations of curves M

Starting with the curve $y = f(x)$

Translate $\binom{0}{a}$ for $y = f(x) + a$

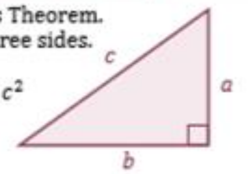
Translate $\binom{-a}{0}$ for $y = f(x + a)$

Stretch parallel to y axis, stretch factor k for $y = kf(x)$

Stretch parallel to x axis, stretch factor $\frac{1}{k}$ for $y = f(kx)$

Right angled triangles NM

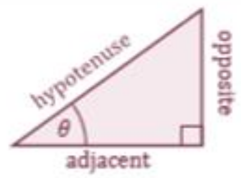
Pythagoras Theorem.
Links all three sides.
No angles.
$a^2 + b^2 = c^2$



Trigonometry.
Links two sides and one angle.
SOH | CAH | TOA

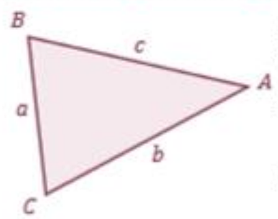
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Use "2ndF" or "SHIFT" key to find a missing angle



The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Advanced trigonometry M



A is opposite *a*
B is opposite *b*
C is opposite *c*

Sine Rule
Use if you are given an angle-side pair

Missing side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Missing angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Cosine Rule
Use if you can't use the sine rule

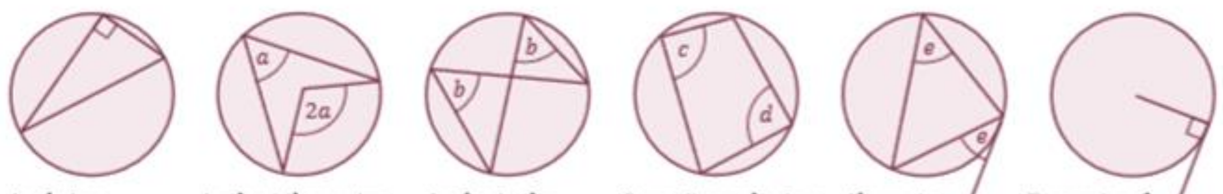
Missing side: $a^2 = b^2 + c^2 - 2bc\cos A$

Missing angle: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Special values of sin, cos, tan
Learn (or be able to find without a calculator)...

θ°	sinθ°	cosθ°	tanθ°
0	0	1	1
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	

Circle theorems M



Angle in a semicircle is 90°

Angle at the centre is double the angle at the circumference

Angles in the same segment are equal

Opposite angles in a cyclic quadrilateral total 180°

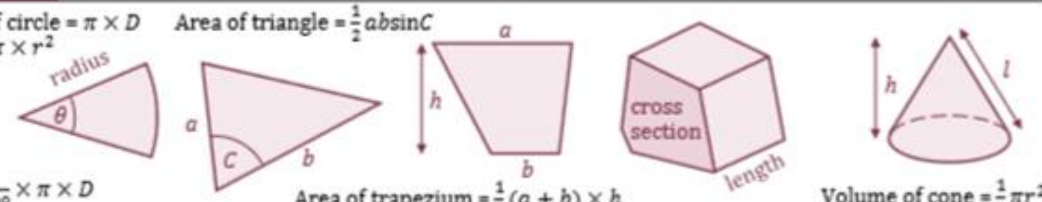
Alternate segment theorem

Tangent and radius are perpendicular

Areas and volumes NM

Circumference of circle = $\pi \times D$
Area of circle = $\pi \times r^2$

Area of triangle = $\frac{1}{2}ab\sin C$



Arc length = $\frac{\theta}{360°} \times \pi \times D$

Area of sector = $\frac{\theta}{360°} \times \pi \times r^2$

Area of trapezium = $\frac{1}{2}(a + b) \times h$

Volume of prism = area of cross section × length

Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone is πrl
[note the right angled triangle with sides r, l and h to find either l or h]

Transformations M

Reflection
• Line of reflection
Translation
• Vector

Rotation
• Centre of rotation
• Angle of rotation
• Clockwise or anticlockwise

Enlargement
• Centre of enlargement
• Scale factor (if −1 < SF < 1 the shape will get smaller).

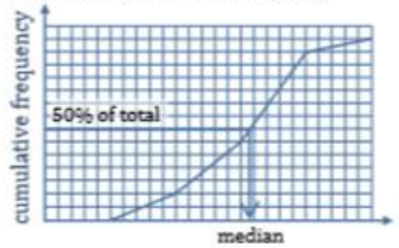
Similar shapes NM

Ratios in similar shapes and solids:

• Length/perimeter	$1:n$	$a:b$
• Area	$1:n^2$	$a^2:b^2$
• Volume	$1:n^3$	$a^3:b^3$

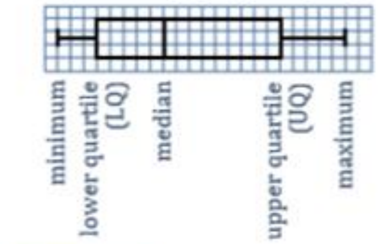
Cumulative frequency NM

Plot points at the **upper end** of each interval. May not start at $(0, 0)$



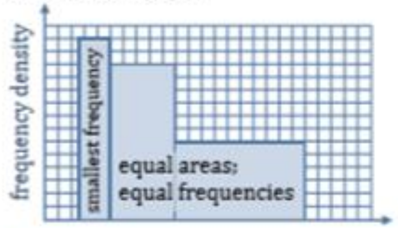
Box plots NM

Interquartile range (IQR) = UQ − LQ

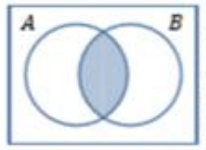


Histograms NM

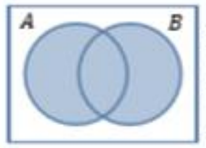
Frequency = frequency density multiplied by class width. This means that bars with the same frequency have the same area.



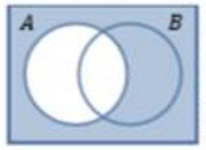
Venn diagrams NM



$A \cap B$
Intersection of *A* and *B*.
Elements in *A* **and** in *B*



$A \cup B$
Union of *A* and *B*
Elements in *A* **or** *B* **or** both



A'
Complement of *A*
Elements which are **not** in *A*

Probability rules M

Multiply for independent events

➔ **P(6 on dice and H on coin)**

$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Add for mutually exclusive events

➔ **P(5 or 6 on dice)**

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Apply these rules to tree diagrams